57. $xy + 2x - y = 0$

$x\frac{dy}{dx} + y + 2 - \frac{dy}{dx} = 0$

$x\frac{dy}{dx} - \frac{dy}{dx} = -y - 2$

$\frac{dy}{dx}(x-1) = -y - 2$

$\frac{dy}{dx} = -\frac{y+2}{x-1}$

tangent $= -\frac{y+2}{x-1}$

normal $= \frac{x-1}{y+2}$

$2x + y = 0$

$y = -2x$

$-2 = \frac{x-1}{y+2}$

$-2y - 4 = x - 1$

$-2y - 3 = x$

$(-2y-3)(y) + 2(-2y-3) - y = 0$

$-2y^2 - 3y - 4y - 6 - y = 0$

$-2y^2 - 8y - 6 = 0$

$y = \frac{8 \pm \sqrt{64-48}}{-4}$

$\frac{8 \pm \sqrt{16}}{-4}$

$\frac{8 \pm 4}{-4}$

$y = -1, -3$

$x(-1) + 2x - (-1) = 0$

$-x + 2x + 1 = 0$

$x = -1$

$(-1, -1)$

$(3, -3)$

$x(-3) + 2x - (-3) = 0$

$-3x + 2x + 3 = 0$

$-x = -3$

$x = 3$

$y = m(x - x_1) + y_1$

$y = -2(x+1) - 1$

$y = -2x - 3$

$y = -2(x-3) - 3$

$y = -2x + 3$

Test

- limit definition of derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- continuous vs. differentiable (corners, cusps, vert. tangents) p. 109
- graphs of $f(x)$ and $f'(x)$
- tangent + normal lines
- rules of differentiation (power, product, quotient, chain, trig functions)
- implicit differentiation
- dist, vel., accel.
- higher order derivatives
- NDER on calc.

$y = 8x^2 + 1$

$y = 16x$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$8(x^2 - a^2)$

$$\frac{8x^2 + 1 - (8a^2 + 1)}{x - a} \Rightarrow \frac{8x^2 - 8a^2}{x - a} = \frac{8\cancel{(x - a)}(x + a)}{\cancel{x - a}}$$

$= 8(x + a)$

$$\lim_{x \to a} 8(x + a) = 8(a + a)$$

$= 8(2a)$

$\boxed{= 16a}$

Homework

→ Study for test

→ Weekly Review 4
(Due tomorrow)

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571

2 : { 1 : integral, 1 : answer }

(b) $H(6) - R(6) = -2.924$,
so the level of heating oil is falling at $t = 6$.

1 : answer with reason

(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026

3 : { 1 : limits, 1 : integrand, 1 : answer }

(d) The absolute minimum occurs at a critical point or an endpoint.

$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$$125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.

3 : { 1 : sets $H(t) - R(t) = 0$, 1 : volume is least at $t = 11.318$, 1 : analysis for absolute minimum }

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.	1 : shows that $R(6) > 0$
(b) $R'(6) = -1.913$ Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.	$2 : \begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$
(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$ To the nearest whole number, there are 964 mosquitoes.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$ $R(t) > 0$ on $0 < t < 2.5\pi$ $R(t) < 0$ on $2.5\pi < t < 7.5\pi$ $R(t) > 0$ on $7.5\pi < t < 31$ The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$. $1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$, There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.	$4 : \begin{cases} 2 : \text{absolute maximum value} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : \text{analysis} \\ \quad 1 : \text{computes interior} \\ \quad\quad \text{critical points} \\ \quad 1 : \text{completes analysis} \end{cases}$

13) $y = x^4 - 2x^2 + 1$
$y' = 4x^3 - 4x$

$y'(0) = 4(0)^3 - 4(0)$
$= 0$

$y'(1) = 4(1)^3 - 4(1) = 0$
$y'(-1) = 4(-1)^3 - 4(-1) = 0$

$y(0) = (0)^4 - 2(0)^2 + 1$
$= 1$

$y(1) = (1)^4 - 2(1)^2 + 1$
$= 1 - 2 + 1 = 0$

$y(-1) = -(1)^4 - 2(-1)^2 + 1$
$= 1 - 2 + 1 = 0$

The tangent is horizontal at $x = -1, 0, 1$
First I found the derivative of the function, which is the equation for the slopes. Then I found when $y' = 0$ because is where the slope is zero and the tangent is horizontal to the x-axis.

+4

11) $y = \frac{x^2}{6-8x}$ $\frac{u}{v}$ $\frac{[v \cdot u'] - [u \cdot v']}{v^2}$

$\frac{[(6-8x)(2x)] - [(x^2)(-8)]}{(6-8x)^2}$ $12x - 16x^2 + 8x^2$

+4

$\boxed{\frac{12x - 8x^2}{(6-8x)^2}}$

#13. The graph of $y = x^4 - 2x^2 + 1$ will have horizontal tangents when

$f'(x) = 0$

$f'(x) = 4x^3 - 4x$ (found by the power rule, see problem 9)

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -4 & 0 \\ & & 4 & 4 & 0 \\ \hline & 4 & 4 & 0 & 0 \end{array}$$

Zero! $\Rightarrow (x-1)(4x^2+4x)$

$$\begin{array}{r|rrr} -1 & 4 & 4 & 0 \\ & & -4 & 0 \\ \hline & 4 & 0 & 0 \end{array}$$

Zero! $= (x-1)(x+1)(4x)$

Since there is no vertical shift of the function, $x = 0$ is also a 0, so

$\boxed{f(x) \text{ will have horizontal tangents at } x = -1,\ 0,\ \text{and } 1}$

+4

2. $y = x - x^2$ $\quad (-2, -6)$

$y' = 1 - 2x$ → substitute -2 for x to find instantaneous slope when x=-2

$y' = 1 - 2(-2)$

$y' = 1 + 4$

$y' = 5$ → slope is 5 when x=-2

$y = 5(x - x_1) + y_1$ ← substitute x_1, y_1 for point $(-2, -6)$

+4

tangent line $\boxed{y = 5(x+2) - 6}$

↳ normal line goes through the same point, but with a perpendicular slope. We find this by using the negative reciprocal of the slope of the tangent line. $\frac{5}{1} \to -\frac{1}{5}$

Substitute (-2, -6) → $y = -\frac{1}{5}(x - x_1) + y_1$

$\boxed{y = -\frac{1}{5}(x + 2) - 6}$

normal line